Finalizing Equations Of Motion: Thrust Inputs from Propellers

This post explains how we determine propeller thrust and drag factors for our quadcopter project.

The last couple posts have been working-out the sum-of-torques on our quad-copter. The first, “unforced” model considers the gyroscopic effect of the total air-frame. That was the first, “equations of motion” post a few weeks ago.

Next we looked at the torque induced by the gyroscopic effects of the spinning propellers in the last post.

Now we’ll consider the propeller drive inputs.

Propeller Drive Control Input

I started the, “equations of motion” with the sketch below, so we’ll use it again here as we model the command inputs from the propellers: F_{1-4} .

ref_frames

Torque About Body X-axis

Assume the length of the arm from the center of the quadcopter to the propeller is, ‘l’ then…

$$\tau_x = F_2 \cdot l - F_4 \cdot l$$

The, ‘F’ terms are thrust forces. We know from an earlier post that this thrust from a propeller is going to be a function of propeller speed squared: Ω^2 :

$F = b \cdot \Omega^2$

Where, ‘b’ is a, “thrust factor” we need to determine. After you watch the video below you’ll understand how we get…

$b = \frac{C_t \cdot \rho \cdot D^4}{4\pi^2}$

where…

$$\begin{aligned}&C_t = experimentally\ determined\ propeller\ thrust\ coef.\\&\rho\ = air\ density.\\&D = propeller\ diameter.\end{aligned}$$

our torque about the X-axis equation above becomes…

$\tau_x = bl(\Omega_2^2 - \Omega_4^2)$

Torque About Body Y-axis

$\tau_y = F_1 \cdot l - F_3 \cdot l$

By the same logic above for the X-axis torque, this Y-axis torque becomes…

$\tau_y = bl(\Omega_1^2 - \Omega_3^2)$

Torque About Body Z-axis

The torque about the Z-Axis is proportional to the squared angular velocities of the propellers. We subtract the squared counter-clockwise props from the clockwise props to lump them. The proportionality constant (the multiplier) is going to be a, “drag factor”: d.

See the video for an explanation on how we get ‘d’ from the experimentally determined, “power coefficient” for a candidate propeller.

$$\tau_z = d \cdot (\Omega_2^2 + \Omega_4^2 - \Omega_3^2 - \Omega_1^2)$$

$$d = \frac{C_P \cdot \rho \cdot D^5}{8\pi^3}$$

$C_P = experimentally\ determined\ propeller\ power\ coef.$
$\rho\ =\ air\ density.$
$D = propeller\ diameter.$

Deriving the, ‘b’ and, ‘d’ terms above

‘b’ is our thrust factor and, ‘d’ is our drag factor. They are arrived at through experimentally determined thrust and power coefficients for our propeller.

An excellent resource is offered by the University of Illinois and Urbana-Champaign: the UIUC Propeller Data Site. There are wind-tunnel data for a variety of hobby-craft propellers. The following video explains how we get our, “factors” for our equations of motion from the experimental, “coefficients”…

Propeller Thrust Coefficient

I am going to estimate my thrust coefficient from the, “static” data because a quadcoptor is not going to travel very fast along the axis of the propellers. I will likely select a propeller model included in the UIUC data sets. My preliminary estimate for the static thrust coefficient after looking at a number of experimental data sets is…

$$C_t = 0.1$$

From this I can calculate, ‘b’ as described above.

Propeller Drag Coefficient

Remember the motor+gearbox+propeller post from a few months ago (already!)? There I was looking at now to map the torque load of the propeller back through the gearbox to the motor shaft to get all the parameters correct in modelling the DC motor drive. In that post I write…

"Aerodynamic torque for rotor R: $D = 2.91\times10^{-7}\frac{Nms^2}{rad^2}$ *(page 9, Gafvert) for what looks like a comparable hobby-sized propeller lab set-up".* Until now, I had no idea how this term was derived.

Confusion…

It was a challenge for me to find a propeller, "drag coefficient" explicitly stated in my references. Then I read, "While an airfoil can be characterized by relations between *angle of attack*, *lift coefficient* and *drag coefficient*, a propeller can be described in terms of *advance ratio*, *thrust coefficient*, and *power coefficient*" here. That statement cleared some fog.

Next I found an online question from a person facing the same confusion I was experiencing. The question married the above quote to the same UIUC data I was looking at. The dialog there ultimately gets to a torque term resolved from the UIUC, "power coefficient".

From there I made sense of it and as expressed in the video above. I derive my, 'd' term from the experimental, "power coefficient" as explained in the video. As the video explains, I'm estimating…

$$C_p = 0.7$$

For the APC Carbon Fiber 7.8-inch propeller, if we plug-in numbers we calculate, 'd' as…

$$d = \frac{C_p\cdot\rho\cdot D^5}{8\pi^3} = \frac{(0.7)(1.23\frac{kg}{m^3})(0.175m)^5}{8\pi^3} = 5.65\times10^{-7}Nms^2$$

This is close to the number from *page 9 of Gafvert* above. I don’t know what specific propeller he is referencing, but we know it is in the family of these hobby propellers. I’m satisfied with this method for arriving at our drag factor from the UIUC data.

Conclusion…

We have our thrust and drag, “factors” for our propellers (in control-system parlance we can think of these as, “gain” terms).

When we get into the design iterations we’ll likely try other propellers with different coefficients. We can revisit the UIUC data and compute a range of, ‘b’ and, ‘d’ terms as we did above. This is how we can assess propeller selection impact on performance of our quadrotor.