Brushless DC (BLDC) Motor Driven Propeller Model

Simple DC Motor model given below. Chan and Woo recommend BLDC be modeled as simple DC motor. The BLDC implements commutation electronically instead of physically but otherwise dynamically similar to a brushed DC motor. This is a motor model you will find in any dynamics textboox.

1) $L_a \cdot \frac{di_a}{dt} + R \cdot i_a + e_b = e_b$

2) $J \frac{d^2\theta}{dt^2} + T_d = T$

Where...
$L_a =$ armature inductance (H)
$i_a = Armature\ Current \cdot (amps)$
$R =$ armature resistance (ohms)
$e_b =$ back emf (V)
$e_a =$ input voltage applied to armature (V)
J = Load referred to motor shaft ($kg \cdot m^2$)
T = Torque output ($N \cdot m$)
$T_d =$ propeller, gearbox, and motor drag ($N \cdot m$)

Assume constant field current and reduce torque to a function of armature current

$T = K_1 \cdot i_a$ where K_1 is the motor constant

Back emf is given as

$e_b = K_b \frac{d\theta}{dt}$ where K_b is the back emf constant

For small motor assume low inductance, so $L_a = 0$, **which after rearrangement of 1) gives...**

$$i_a = \frac{e_a}{R} - \frac{K_b}{R} \cdot \frac{d\theta}{dt}$$

Substituting this into 2) above...

$$J\frac{d^2\theta}{dt^2} + T_d = K_1 \cdot i_a = K_1 \cdot \left(\frac{e_a}{R} - \frac{K_b}{R} \cdot \frac{d\theta}{dt} \right)$$

Here we pick-up propeller system specific terms from the Bouabdallah paper. An equation for the total propeller, gearbox, and motor drag is given by...

$$T_d = \frac{d}{\eta r^3 J} \cdot \omega$$

Where...

d = drag factor

η = gearbox efficiency

r = gearbox reduction ratio

$\omega = \frac{d\theta}{dt}$

Substituting and rearranging we arrive at...

$$\frac{d\omega}{dt} = -\frac{K_1 \cdot K_b}{R \cdot J} \cdot \omega - \frac{d}{\eta r^3 J} \cdot \omega^2 + \frac{K_1}{R \cdot J} \cdot u$$

where u is motor input to armature voltage $(e_a\ above)$

We want to linearize about a propeller operating speed ω_0 to gain an equation for $\frac{d\omega}{dt}$

$$f := (\omega) \rightarrow -\frac{K_1 \cdot K_b}{R \cdot J} \cdot \omega - \frac{d}{\eta r^3 J} \cdot \omega^2 + \frac{K_1}{R \cdot J} \cdot u$$

$$\frac{d\omega}{dt} = f(\omega_0) + \left.\frac{\partial f}{\partial \omega}\right|_{\omega = \omega_0} \cdot (\omega - \omega_0) \quad + \textit{higher terms we will neglect}$$

$$f(\omega_0) = -\frac{K_1 K_b \omega_0}{R J} - \frac{d\, \omega_0^2}{\eta r^3 J} + \frac{K_1 u}{R J}$$

$$\left.\frac{\partial f}{\partial \omega}\right|_{\omega = \omega_0} = -\frac{K_1 K_b}{R J} - \frac{2\, d\, \omega_0}{\eta r^3 J}$$

$$g := \omega \rightarrow$$

$$-\frac{K_1 K_b}{R J} - \frac{2 d \omega}{\eta r^3 J}$$

So the Taylor's Series expansion yields a linear equation for $\frac{d\omega}{dt}$ about operating point ω_0 as follows

$$\frac{d\omega}{dt} = -\left(\frac{K_1 K_b}{R J} + \frac{2 d \omega_0}{\eta r^3 J}\right) \cdot \omega + \frac{K_1}{R \cdot J} \cdot u + \frac{d \omega_0^2}{\eta r^3 J}$$

(1.1)

Let...

$$A = -\left(\frac{K_1 K_b}{R J} + \frac{2 d \omega_0}{\eta r^3 J}\right)$$

$$B = \frac{K_1}{R \cdot J}$$

$$C = \frac{d \omega_0^2}{\eta r^3 J}$$

(1.2)

Therefore we arrive at a linearized equation we can design a propeller speed controller loop for.
We will need to determine method for sliding the A and C constants over the operating speed range.
This will also likely require compensator adjustment for the revised constants.

$$\frac{d\omega}{dt} = A \cdot \omega + B \cdot u + C$$