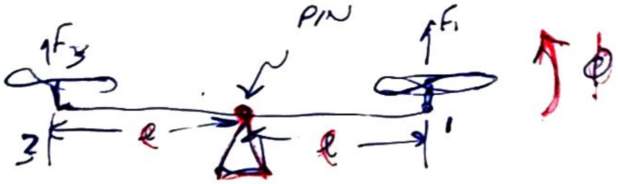
8/24 LINEARIZED, SIMPLIFIED I.O. MODEL TO UNDERSTAND MAJOR CONTRIBUTORS TO QUADCOPTER DYNAMIC BEHAVIOR

1) ASSUME "LEVEL" CONFIGURATION (HOVER)

- NEGLECT BODY GYROSCOPIC EFFECTS
- NEGLECT PROPELLER GYROSCOPIC EFFECTS

2) CONSIDER A SINGLE CROSS-BODY PROPELLER PAIR PINNED AT C.G.



$$I\ddot{\phi} = F_1 l - F_3 l = \frac{l}{I} u_1$$

$$u = \boxed{b}\left(\omega_1^2 - \omega_2^3\right)$$

where ω_1 AND ω_3 ARE ROTOR SPEEDS

$\Rightarrow$ Recall $\dot{\omega} = -A\omega(t) + Bv(t) + C$

EXAMINE RELATIVE MAGNITUDE OF A, B, + C

$$\frac{d\omega}{dt} = -\left(\frac{K_1 K_b}{RJ} + \frac{2 d\omega_0}{\eta r^3 J}\right)\omega(t) + \frac{K_1}{RJ}v(t) + \frac{d\omega_0^2}{\eta r^3 J}$$

$$f(\omega_0) = -\frac{k_1 k_b \omega_0}{RJ} - \frac{d\omega_0^2}{\eta r^3 J} + \frac{k_1}{RJ} u$$

$$\left.\frac{\partial f}{\partial \omega}\right|_{\omega=\omega_0} = -\frac{k_1 k_b}{RJ} - \frac{2 d \omega_0}{\eta r^3 J}$$

$$\frac{d\omega}{dt} = f(\omega_0) + \left.\frac{\partial f}{\partial \omega}\right|_{\omega=\omega_0} (\omega - \omega_0)$$

$$= -\frac{k_1 k_b \omega_0}{RJ} - \frac{d\omega_0^2}{\eta r^3 J} + \frac{k_1}{RJ} u$$

$$+ \left[-\left(\frac{k_1 k_b}{RJ} + \frac{2 d \omega_0}{\eta R^3 J} \right) \right] (\omega - \omega_0)$$

$$= \cancel{-\frac{k_1 k_b \omega_0}{RJ}} - \frac{d\omega_0^2}{\eta r^3 J} + \frac{k_1}{RJ} u - \left(\frac{k_1 k_b}{RJ} + \frac{2 d \omega_0}{\eta R^3 J} \right) \omega + \cancel{\frac{k_1 k_b}{RJ} \omega_0} \frac{2 d \omega_0^2}{\eta R^3 J}$$

$$\Rightarrow \frac{d\omega}{dt} = -\overbrace{\left(\frac{k_1 k_b}{RJ} + \frac{2 d \omega_0}{\eta r^3 J} \right)}^{A} \omega + \overbrace{\frac{K_1}{RJ}}^{B} V(t) + \overbrace{\frac{d \omega_0^2}{\eta r^3 J}}^{C}$$

- UNFORCED, WE SEE $\frac{d\omega}{dt}$ DIMINISHES ACCORDING TO THE "A" TERM.
- FORCED WE SEE B INCREASES $\frac{d\omega}{dt}$ ACCORDING TO "B" TERM AND APPLIED VOLTAGE V: V APPLIED TO MOTOR ACCELERATES THE PROPELLER
- "C" TELLS US THAT THERE IS AN ω_0-RELATED TERM THAT "MAINTAINS" OR HOLDS-UP $\frac{d\omega}{dt}$ FOR HIGH NOMINAL $\omega = \omega_0$... THIS LOOKS TO HAVE SIMILAR EFFECT AS THE SECOND TERM IN "A".

See companion PDF for explanation of dropping the 'C' term to simplify the propeller drive differential equation.

$$\dot{\omega} = -A\,\omega(t) + B\,v(t)$$

$$\mathcal{L} \Rightarrow s\,\omega(s) = -A\,\omega(s) + B\,v(s)$$

$$\omega(s)(s+A) = B\,v(s)$$

$$\omega(s) = \frac{B}{s+A}\,v(s)$$

SO OUR CONTROL INPUT U BECOMES...

$$U = b(\omega_1^2 - \omega_3^2) = b\left[\frac{B^2}{(s+A)^2}V_1(s)^2 - \frac{B^2}{(s+A)^2}V_3(s)^2\right]$$

$$U = \frac{bB^2}{(s+A)^2}\underbrace{\left(V_1(s)^2 - V_3(s)^2\right)}$$

LET THIS DIFFERENCE BE DENOTED $\mathcal{V}(s)$

$$\ddot{\phi} = \frac{l}{I_x}U \overset{\mathcal{L}}{\Rightarrow} s^2\phi(s) = \frac{l}{I_x}\frac{bB^2}{(s+A)^2}\mathcal{V}(s)$$

$$\frac{\phi(s)}{\mathcal{V}(s)} = \frac{\frac{lbB^2}{I_x}}{s^2(s+A)^2} \leftarrow K$$

$$\Rightarrow \frac{\phi(s)}{\mathcal{V}(s)} = \frac{K}{s^2(s+A)^2}$$

IS OUR TRANSFER FUNCTION FROM DRIVE VOLTAGE DIFFERENCE TO ANGULAR OUTPUT FOR ONE ARM OF THE QUAD CROSS